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Comprehending the Concept of Functions

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Abstract

Constructivist theory of learning posits that students construct their own learning which may differs from formal mathematics. The role of constructivist teacher is to understand student's conceptions, identifying errors or difficulties associated with the process of learning before teaching can begin. This paper reports on a study that investigated Malaysian secondary students' comprehension of the concept of functions. The students were individually interviewed while solving problems in four seperate sessions. Qualitative analysis of their responses indicated three main types of difficulties: symbols of f(x), connecting f(x) with graph and formal set theoretic definition of function. Critical protocols from interviews sessions illustrated the stumbling block faced by these students in comprehending this important concept. The difficulties highlighted could serve as a basis towards helping students to modify their understanding of functions to a more sophisticated level of mathematics learning.

***Keywords*:** Concept of function; Readiness; Mathematics; Learning

1. Introduction

Romberg et al. (1989) suggested two disciplines of scientific inquiry that ought to be taken into account when responding to the issues of learning in school. The first is research on how students' learn and secondly is research on how to teach. They stressed that teachers ought to recognize students different ways of learning in order to help students overcome possible difficulties especially when making the transition from intuitive to abstract knowledge such as in mathematics. Students learned by connecting new ideas to prior knowledge, therefore teachers understanding of students' prior knowledge can aid teaching and learning (NCTM, 2000). Shulman (1986) proposed,

> *The essential task of for the teacher, is to appraise, infer, or anticipate these prior cognitive structures to students bring to learning situation, teacher must organize the content of their instruction in terms of those preconceptions, actively working to reveal and transform them when they would interfere with adequate comprehension of the new material to be taught (p.6)*

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Black and William (1998) reviewed about 250 research studies and concluded that students' learning is generally enhanced in classrooms where teachers gave attention to formative assessment in making judgements about teaching and learning.

The concept of function is one of the most important topics in Malaysian secondary school mathematics curriculum. Function plays an important role in algebra and trigonometry which eventually leads to learning of calculus. Various researches on students development and understanding of function concepts has been explored, however they varies in focus, level of students schooling and theoretical perspectives used (Sfard,1995, Kieren,1992, Thompson, 1994, Janvier,1989; Tall & Vinner,1981; Swafford & Langrall, 2000). Cognitive perspectives focus on building relationship between student internal and external representations. Understanding the concept of function from cognitive perspectives implies an ability to make connections between different representations of the concept. However Thompson (1989) argued from constructivist perspectives that the concept of function is not represented by what are commonly called the multiple representations of function,

> *Instead our making connections among representational activities produce a subjective sense of invariance. We do not focus on graphs, expressions, or tables as representations of function but instead focus on them as representations of something that, from students' perspective, is representable, such as aspects of a specific situation (p. 39).*

This study is based on constructivist perspectives in learning. Twelve above average form four students were selected from three schools in Penang (northern state of Malaysia) with the objective to probe students' thoughts processes as they solve problems related to the concept of functions. An intensive interview session was carried out on weekly basis for the duration of two months. This paper however shall only discussed students' difficulties that were observed during each problem solving activities.

2. Methodology

The main sources of data were collected from clinical interviews (Appendix A show selected tasks used). The individual interviews focused on each student thinking, rather than just the written responses. During the interviews, questions like, "Why?" and "How do you do this?" were frequently used. To determine the certainty behind what the student says methods of "repetition" and "counter-suggestion" was employed to help researcher to gain better insight of the situation (Piaget, 1972; Ginsburg,1981).

3. Findings

The findings of this study were mainly based on the qualitative data gathered from the respondents using a developed set of interview tasks which were first transcribed. The analysis of all subjects written work and researcher's notes during the interview sessions were categorized according to themes and analyzed for each subject and across the subjects for each themes. Three main difficulties were observed and classified. There were symbols f(x), relationship between function and graph, and formal definition of function. Each shall be discussed and critical protocols will be provided to highlight the stumbling block faced by students in this study. These results are presented as follows:

3.1 Symbol related to f(x)

Two related aspects of functional symbols indicated greatest obstacles to most of the students in this sample.

3.1.1 Variable x

Students were unaware that x is a variable in the following notation of $f(x)$. Some were observed to retained $f(x)$ in the formula even though x was already given a value. The following excerpt illustrated in Figure 1, shows one of the student work,

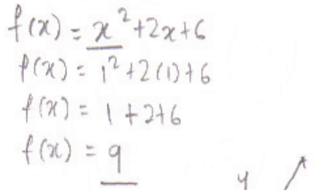

Figure 1

This student replaced x=1, but he only did so on the right hand side while retaining the symbol x on the left. This behaviour seem to indicative of limited understanding of $f(x)$ as a formula and $f(1)$ as a value when $x = 1$.

The difficuty in understanding the symbol x is a variable in $f(x)$, surfaced again when a few students were observed to equate $p(x - 4) = p(x) - 4$. The following excerpt in Figure 2 illustrated student behaviour when solving Task 3: [P: researcher, S. student],

P: Please explain what have you done here?
S: We know.. pf(x) is the same as 2x plus 5….so I replace f(x) ..in p(x) ..and get this...x bring over the other side
P; Why is that 2x now x?
S: x go over that side and 4 bring over here becomes…9
P: x here ? [*researcher pointed to p(**x**-4) written by student]*
S: x bring over the other side ….so.. 2x- x becomes x

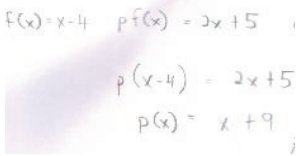

Figure 2

3.1.2 Equations and functions

A few students were not able to differentiate between equations and functions when attempting Task 1. For example, one student thought that only quadratic graphs represents function while linear graphs do not. The following excerpt illustrates this behavior:

P: Why is G2 function?
S: Because it is U shaped..quadratic
P: Can you explain more?
S: (*silence*)
P: Can function take some other shape?
S: *(silence)*
P: Look at G6..G6 what is that ?
S: (*long silence*)......linear equation
P: Please write
S: *(student wrote)* x+2 = 5

This student showed confusion as she struggled to differentiate between functions and equations. This excerpt also suggested that her images of function are primarily of a quadratic form. She did not recognize that linear graph also represent linear function. Instead she wrote linear graph as x+2 = 5. The confusion between the idea of function and equations were notably observed among a number other students in this study.

3.2 Coordinating between f(x) and graf of f(x)

Students' difficulties with graphical contexts were observed as they tried to read the coordinates of Cartesian graph. Relating equations to Cartesian coordinates is not trivial to some of these students. For example, one student struggled when relating equation y = *f(x)* with coordinates (*x*, *f(x)*), while another student read coordinates (*x*, *y*) by looking at the furthest corner of the graph. The following excerpt in Figure 3 illustrates the difficulties faced by one of the student as he tried to relate the f(x) = y to the coordinate (x, y) in Task 3.

P: Could you tell me what is f(1)
S: I cannot see...f(1) (*student drew on graph f*)
P: Where is f(1)?
S: Here (*student showed vertical line that he draw*)

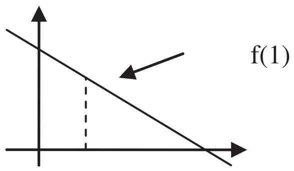

Figure 3

This student was not able to relate y = *f(x)* to its graphical representations and seems unaware that each infinite points on the line represent coordinate (*x*, *y*) or specifically (*x*, *f(x)*). Flexibility is important in mathematics learning and students need to be exposed to varieties of experiences in using notations and symbols related to mathematics symbols. Discussions with teachers in these schools suggested that most of their classrooms activities were related to the topic "graph of functions" separately from 'quadratic functions". Lessons therefore according to them normally start with introduction to formula y = f(x) followed with students constructing of graph by calculating coordinates before plotting each points on the graph.

3.3 Formal definition of function

In Malaysian context, students were introduced to function in form four as formal definitions of a special mapping c two sets. This definition mainly stresses mapping of two sets; each object are maps onto one image. The definition ma seem rather straight forward; however these students in this study portrayed other wise. The following extract showe one student who could not recall the formal definition but uttering an intuitive idea about function.

P: in topic of function what did you learn?
S: a special relation...
P: ok can you give me an example?
S: oh..mother and daughter..a special relation
P: what do you mean special?
S: it is special because.. fixed like that ..we cannot change..to something else

For next task, students were given mapping of f(x) and g(x). They were then requested to calculate fg(x) and illustrate this as mapping of set. All of the students were able to calculate for fg(x). However only two were able to do draw the mappings correctly while the rest displayed several confusions. The following excerpt in Figure 4 showed some of these atypical difficulties.

P: ok....complete the diagram
S: *(students' drew)*

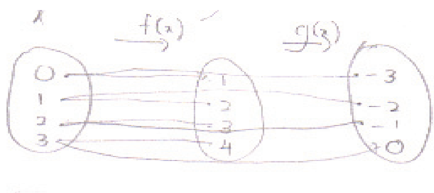

Figure 4

P: Can you find gf(1)
S: (*students' calculation*)
gf(1) = g(x+1) = 2-3 = -1
P: gf(1) can you get from the figure given?..
S: (*student frowned)*
P: Can you get gf(1)..without calculating...but refer to the figure?
S: *(student did not responded.*)
P: can you do that?
S: No ..[I] can only calculate.

Despite knowing the definition of function and able to obtain *fg(x)* algebraically, she could not show the direction of the mappings *fg(x)* that she has calculated. She was unable to relate composition of functions *fg(x)* as mapping of *g(x)* followed by *f(g(x)).* A few other students also exhibited a similar difficulties indicating a strong tendencies to recall from their experiences of what was done in the classroom, rather than applying the definition of function directly.

4. Discussion

This paper illustrates some of the challenges and difficulties faced by students in this study as they attempted to solve problems related to the concept of functions. The findings of this study have several implications to classroom instructions. Instructional approach to teaching the topic of functions needs to be improvised. This study has shown that students grapple with the idea of function as similarly observed by many other international studies. Majority of these students has yet to master basic operations of algebra and they seem to be operating superficially with the symbols. Unless the students have attained the flexibility of using algebraic symbolism it would not be easy for them to move ahead beyond procedural learning (Herscovics & Linchevski, 1994). Some of the students in this study also exhibited difficulties using Cartesian graph. Reading coordinates and writing equation of the graph were two main obstacles observed. Since topic of function is introductory topic in additional mathematics subject at form fours, review of algebraic rules including the use of Cartesian coordinates seems to be appropriate and essential before new learning can take place.

The concept of function taught in KBSM is primarily limited to symbolic manipulation (KPM, 2003). There is a need to address how contexts and the representations of functions can be enriched in the classrooms activities. Several educators have highlighted the importance of considering the different representations when constructing mathematical concept. This implies selecting appropriate tasks to in order that students have rich repertoire of functional representations. Textbooks applications hardly offer examples of application which can be found in abundance in everyday life. Shahrir (2000) discussed and recommended that national mathematics curriculum should attempt to include humanistic aspect of mathematics learning in order that mathematics remains accessible to all students. Abstract mathematical concepts must be linked to everyday experiences.

With teacher serving as guide in mathematics classroom students can overcome various learning difficulties in mathematics especially related to the concept of functions. Mathematics teachers, who want to promote learning with understanding, need to be able to design appropriate instructional strategies based on student's prior knowledge and difficulties so as to help students towards a better construction of knowledge. It is fundamental that teachers continue to support and guide students towards more sophisticated learning.

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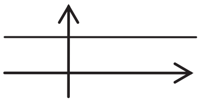
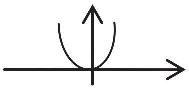
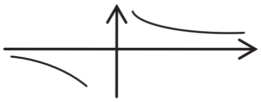
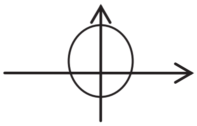
Appendix A

Task 1: Decide which of the graph represent function. Explain why.


G1


G2


G3


G4

Task 2: Given $f(x) = x - 4$ and $g(\mathrm{x}) = \sqrt{\mathrm{x}}$, $\mathrm{x} > 0$

Find *fg(x), fg(1)* b) If $f\,h\,(\mathrm{x}) = x+1$, find $h(x)$. c) If $hf\,(\mathrm{x}) = 2x + 5$, find $h(x)$.

Task 3: Find the value of $gf\,(2)$.